Name: ______________________________

Student ID#: ________________________

Section: ________________________

Midterm Exam 1

Friday, May 3

MAT 21C, Reintjes, Spring 2019

Print names and ID's clearly, and have your student ID ready to be checked when you turn in your exam. Write the solutions clearly and legibly. Do not write near the edge of the paper or the stapled corner. Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. may be used on this exam.

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
6		20 (Extra Points)
Total		100

1.) Compute the limits of the following sequences:
(You can use special limits, such as $\lim_{n\to\infty} \frac{1}{n} = 0$, $\lim_{n\to\infty} \frac{1}{n^2} = 0$,...)

(a) $\lim_{n\to\infty} \left(4 + \frac{1}{n}\right)$,

(b) $\lim_{n\to\infty} \frac{n^2+1}{2n^2+3}$,

(c) $\lim_{n\to\infty} \frac{\sin(n)}{n^2}$, (Hint: Sandwich Theorem)

(d) $\lim_{n\to\infty} \cos\left(\frac{1}{n}\right)$, (Hint: $\cos(x)$ is a continuous function...)

You may use this sheet as scratch paper or to continue any solution (clearly mark the problem).

2.) Determine whether the following series converge or diverge:
(Don't compute their limits, but give reason to your answer!)

(a) $\sum_{n=1}^{3} \frac{1}{n}$,

(b) $\sum_{n=1}^{\infty} \left(\frac{4n}{n^3} + \frac{2}{n^2} \right)$, (Hint: Use that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.)

(c) $\sum_{n=1}^{\infty} e^{-n}$, (Hint: Make comparison based on $0 \le e^{-n} \le \frac{1}{n^2}$.)

(d) $\sum_{n=1}^{\infty} \frac{4n+1}{n}$.

You may use this sheet as scratch paper or to continue any solution (clearly mark the problem).

3.) Determine whether the following series converge absolutely or conditionally:
(Hint: Geometric series formula, Ratio Test, Root Test or Alternating Series Theorem could be useful.)

(a) $\sum_{n=0}^{\infty} \frac{2}{5^n}$,

(b) $\sum_{n=1}^{\infty} \frac{3^n}{n^n}$,

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

You may use this sheet as scratch paper or to continue any solution (clearly mark the problem).

4.) (20pts) Determine the radius of convergence ($0 \leq R \leq \infty$) of the following power series: (Hint: Use Geometric series, Ratio or Root Test!)

(a) $\sum\limits_{n=0}^{\infty} (x-1)^n$,

(b) $\sum\limits_{n=1}^{\infty} \frac{2n}{n+1} x^n$.

You may use this sheet as scratch paper or to continue any solution (clearly mark the problem).

5.) Investigate the Taylor series of $f(x) = e^{-x^3}$ as follows:

(a) Compute the Taylor Series of $f(x) = e^{-x^3}$ at center point $a = 0$!
(b) Determine the radius of convergence ($0 \leq R \leq \infty$) of the resulting Taylor series!
(c) Give reason why the Taylor series agrees with $f(x)$!

(Hint: You may use that $e^x = \sum\limits_{n=0}^{\infty} \frac{x^n}{n!}$.)

You may use this sheet as scratch paper or to continue any solution (clearly mark the problem).

6.) Extra Points problem (+20pts): Taylor Series of $f(x) = \frac{1}{2-x}$.

(a) Compute the Taylor Series of $f(x) = \frac{1}{2-x}$ at center point $a = 4$!

(b) Determine the radius of convergence of the resulting Taylor series!

(c) Where does the Taylor series agree with $f(x)$? (Give reason!)

(Hint: You may use the geometric series formula $\frac{1}{1-x} = \sum\limits_{n=0}^{\infty} x^n$.)

You may use this sheet as scratch paper or to continue any solution (clearly mark the problem).